

## Performances of Non-linear Smooth Transition Autoregressive and Linear Autoregressive Models in Forecasting the Ringgit-Yen Rate

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### ABSTRAK

Kajian ini membandingkan prestasi peramalan di antara model ketaklinearan Autoregresi Berperalihan Licin (Smooth Transition Autoregressive atau STAR) dengan model linear siri masa iaitu model Autoregresi (Autoregressive atau AR) sebagai peramal kadar pertukaran Ringgit-Yen. Berdasarkan kepada prosedur penentuan kelinearan piawai, kami memperoleh bukti empirik bahawa penyelarasan Ringgit-Yen ke arah keseimbangan pariti kuasa beli (Purchasing Power Parity) adalah secara tidak linear. Dari segi prestasi peramalan, keputusan empirik menunjukkan bahawa model STAR dan AR mempunyai purata mutlak ralat ramalan (mean absolute forecast error atau MAFE), purata mutlak peratus ralat ramalan (mean absolute percentage forecast error atau MAPFE) dan purata punca kuasa dua ralat ramalan (mean square forecast error atau RMSFE) yang lebih kecil jika dibanding dengan model SRW. Keputusan yang diperoleh juga menunjukkan ramalan model STAR lebih baik jika dibandingkan dengan pesaing linearnya, iaitu model AR. Hasil kajian ini adalah konsisten dengan penyelidikan yang memberi penekanan kepentingan membenarkan pelarasan yang tidak linear bagi kadar pertukaran asing ke arah keseimbangan jangka panjang.

### ABSTRACT

This study compares the performance of Smooth Transition Autoregressive (STAR) non-linear model and the conventional linear Autoregressive (AR) time series model in forecasting the Ringgit-Yen rate. Based on standard linearity test procedure, we find empirical evidence that the adjustment of the Ringgit-Yen rate towards its long-run Purchasing Power Parity equilibrium follows a non-linearity path. In terms of forecasting ability, results of this study suggest that both the STAR and AR models exceed or match the performance of SRW model based mean absolute forecast error (MAFE) mean absolute percentage forecast error (MAPFE) and mean square forecast error (RMSFE). The results also show that the STAR model outperforms the AR model, its linear competitor. Our finding is consistent with the emerging line of research that emphasised the importance of allowing non-linearity in the adjustment of exchange rate toward its long run equilibrium.

### INTRODUCTION

In 1926, Yule first formally introduced the time series model in the form of autoregressive (AR) model, which assumes that the future values of a variable depend solely on its historical values. Since then, time series analysis has been viewed as a powerful forecasting tool. In the past two decades or so the theory of time series

econometrics is progressing rapidly (Montgomery *et al.* 1990). As the methodology progresses, the issue of non-linearity was incorporated into the analysis of time series. Smooth Transition Autoregressive (STAR) model is one of the most recent models developed under this concept. The STAR model is a non-linear time series model that allows the variable



under investigation to move within two different state spaces with a smooth transition process. STAR offers an alternative to the modelling of time series variables that exhibit non-linearities.

In our review of the literature, we found that the application of the STAR model in empirical works is still very limited, and its forecasting performance particularly with reference to exchange rates has yet to be determined. Taylor and Peel (2000), Sarno (2000a) and Baum *et al.* (2001) are among the first to demonstrate the usefulness of the STAR in modelling exchange rate dynamics. However, all these authors did not evaluate the forecasting performance of the model. Hence, the main objective of this study is to examine the applicability of the STAR model to the Malaysian Ringgit against the Japanese Yen (RM/YEN). In addition, the present article compares the forecasting performance of the STAR and AR models by using the simple random walk (SRW) model as the yardstick of comparison. This paper extends previous studies by evaluating the forecasting performance of the studied models using mean absolute forecast error (MAFE), mean absolute percentage forecast error (MAPFE) and root mean square forecast error (RMSFE).

We chose to apply the model to the exchange rate for the following reasons: First, the bulk of the literature shows that structural models failed to outperform a simple random walk (SRW) model, and attempts by analysts using more elaborate models have also failed to improve the forecast performances at short or long run horizons significantly. Second, several authors have argued that the failure of existing exchange rate models to yield superior forecast is because these models ignore the non-linearity adjustment of exchange rates towards its equilibrium value (Micheal *et al.* 1997; Taylor and Peel 1997; Sarno 2000a,b; Coakley and Fuertes 2001). These studies also argued that the classical unit root tests may not be able to detect mean reverting behaviour of exchange rate if the variable is a stationary non-linear process. Finally, in applying this model to the Ringgit, we intend to broaden our understanding on the appropriateness of the STAR model as a forecasting tool in the currency markets.

This study models the adjustment process of the deviations of Yen-based Ringgit movement from its fundamental equilibrium as determined by the Purchasing Power Parity (PPP) hypothesis. Simply, the PPP hypothesis postulates that the nominal exchange rate is given by the ratio of the domestic and foreign price levels. It states that exchange rates should tend to equalize prices for identical goods in different countries. Recent studies based on careful application of time series econometrics methods, are more supportive of the mean reverting behaviour of exchange rates towards the long-run PPP equilibrium value (M. Azali *et al.* 2001; Baum *et al.* 2001)<sup>1</sup>.

To anticipate our results, we find that RM/YEN rate adjusts in a non-linear fashion towards its long-run equilibrium path using the Lagrange Multiplier (LM)-type test developed by Teräsvirta (1994). Unlike most of the earlier studies, we find that both the STAR and AR models outperform a random walk forecast for nominal RM/YEN rate. In addition, the empirical results suggest that the non-linear STAR model performs better than the linear AR model in the out-sample forecasts. The set out of the paper is organised as follows. The first two sections offer a brief review on the development of STAR models and a discussion on the data used in the analysis. The section that follows immediately describes the linearity test and the test results. We report and interpret the results of forecast accuracy comparison just before offering some concluding remarks in the last section.

## THE STAR MODELS

The origin of the non-linear Smooth Transition Threshold Autoregressive or just Smooth Transition Autoregressive (STAR) model could be traced back to the Threshold Autoregressive (TAR) model first proposed by Tong in 1977 (see Tong and Lim 1980). The TAR model assumes that a variable has different behaviour within different regimes. The basic idea underlying the TAR model is piecewise linearisation of non-linear models over the state space by the introduction of the thresholds. An example of the TAR model is the Self-excited TAR (or SETAR) model, which assumes that a

<sup>1</sup> The stylized fact that emerged from this literature is that exchange rate adjusts non-linearly towards its long-run PPP equilibrium (Mahajan and Wagner 1999; Sarno 2000a; Baum *et al.* 2001 and Coakley and Fuertes 2001).



variable (say, exchange rate),  $y_t$  is a linear autoregression within regime, but may move between regimes depending on the value taken by a lag of  $y_t$ , say  $y_{t-d}$  where  $d$  is known as delay parameter. For two-regime case ( $q = 2$ ) where  $y_t$  follows an AR ( $p_1$ ) process in the one regime and AR ( $p_2$ ) process in the other, the SETAR (2;  $p_1, p_2$ ) representation of can be written compactly as:

$$y_t = \beta_0 + \sum_{i=1}^p \beta_i y_{t-i} + \varepsilon_{1t} + I_{t-d}(r) \left[ \beta_0^* + \sum_{i=1}^{p_2} \beta_i^* y_{t-i} + \varepsilon_{2t} - \varepsilon_{1t} \right] \quad (1)$$

where  $I_t(r) = 1$  if  $y_t > r$  and 0 otherwise is the threshold. For  $h = 1, 2$ ,  $\varepsilon_{ht} \sim G(0, \sigma_h^2)$  where  $G(\cdot)$  may be a Gaussian distribution but this is not necessarily the case.  $\beta_i$  for  $i = 0, \dots, p_1$  and  $\beta_i^*$  for  $i = 0, \dots, p_2$  are parameters to be estimated.

The introduction of non-linear time series model such as SETAR model is motivated by the fact that linear time series model should give place to a much wider class of models if we were to gain more understanding into the more complicated phenomena such as limit cycles, time irreversibility, amplitude-frequency dependency and jump resonance (Tong and Lim 1980). Since its introduction, few attempts have been made in applying and validating the SETAR mode, and hence the usefulness of the model in empirical work is yet to be determined. For instance, Diebold and Nason (1990) point out that there is no guarantee that SETAR model will perform better than linear AR model. A similar view is expressed in Clements and Smith (1997), where they note that neither in-sample, nor the rejection of null of linearity in a formal test in favour of non-linearity guarantees that SETAR predicts more accurately than AR models.

The deficiency in SETAR is deemed due to the unrealistic fixed threshold in the model. The fixed threshold of SETAR model is later replaced with a smooth function and thus leads to the formation of STAR model in the early 1990s. STAR model allows the variable under study to alternate between two different regimes with a smooth transition function between these regimes, so that there can be a continuum of states between extreme regimes. STAR

representation is given by (Teräsvirta and Anderson 1993):

$$y_t = \beta_0 + \sum_{i=1}^p \beta_i y_{t-i} + \left( \beta_0^* + \sum_{i=1}^{p_2} \beta_i^* y_{t-i} \right) F(y_{t-d}) + \varepsilon_t \quad (2)$$

where  $y_t$  is mean-corrected,  $\beta_0$ , and  $\beta_0^*$  are constants,  $\beta_i$  and  $\beta_i^*$ ,  $i = 1, \dots, p$  are autoregressive parameters,  $F(\cdot)$  is the transition function depending on the lagged level,  $y_{t-d}$ , where  $d$  is known as the delay length or delay parameter, and  $\varepsilon_t$  is a white noise with zero mean and constant variance  $\sigma_\varepsilon^2$ .

For the application of STAR model, Granger and Teräsvirta (1993) have proposed exponential function as one of the plausible transition functions, thus resulting in the exponential STAR or ESTAR model. The exponential function is defined as:

$$F(y_{t-d}) = 1 - \exp\left(-\gamma^2(y_{t-d} - u)^2 / \hat{\sigma}_{y_t}^2\right) \quad (3)$$

where  $\gamma^2$  is the unstandardized transition parameter,  $u$  is the equilibrium or threshold value of the mean corrected  $y_t$  series and hence  $E(u) = 0$ , and  $\hat{\sigma}_{y_t}^2$  is the estimated variance of  $y_t$ .

Note that  $\gamma^2 / \hat{\sigma}_{y_t}^2$  is the standardized transition parameter and the speed of transition between the two regimes is positively related to the value of the transition parameter  $\gamma^2$ . In other words, higher values of  $\gamma^2$  imply much faster speed of transition. Taylor and Peel (2000) use a version of transition function,  $F(\cdot)$  with  $\hat{\sigma}_{y_t}^2 = 1$ . Nevertheless, Granger and Teräsvirta (1993, p. 124) argue that scaling the exponential term by the sample variance speeds the convergence and improves the stability of the non-linear least squares estimation algorithm. It also makes it possible to compare estimates of transition parameter across equations.

The exponential transition function is bounded between zero and one. Judging from Equation 3, when  $y_{t-d}$  equals its equilibrium value  $u$  or when  $\gamma^2 / \hat{\sigma}_{y_t}^2$  goes to zero,  $F(\cdot) = 0$  and Equation 2 reverts to a standard linear AR( $p$ ) representation:



$$y_t = \beta_0 + \sum_{i=1}^p \beta_i y_{t-i} + \varepsilon_t \quad (4)$$

In such case, the conventional restriction of

$$\sum_{i=1}^p \beta_i < 1 \text{ applies so that } y_t \text{ is mean-reverting.}$$

For extreme deviations from the fundamental equilibrium,  $F(\cdot) = 1$  (when  $\gamma^2 / \hat{\sigma}_y^2$  approaches infinity), and Equation 2 becomes a non-linear AR ( $p$ ) model:

$$y_t = (\beta_0 + \beta_0) + \sum_{i=1}^p (\beta_i + \beta_i^*) y_{t-i} + \varepsilon_t \quad (5)$$

If the non-linear model as in Equation 5 is the

correct specification, it is expected that  $|\sum_{i=1}^p \beta_i| \leq 1$

such that  $y_t$  may exhibit unit root behaviour but the requirement for global stability is that

$$|\sum_{i=1}^p (\beta_i + \beta_i^*)| < 1 \text{ must be met.}$$

The exponential  $F(\cdot)$  allows a smooth transition between two regimes symmetry adjustment for deviations above and below the fundamental equilibrium. This function is considered suitable for the non-linear modelling of exchange rate as it has a number of attractive properties. For instance, it can capture the symmetrical response to positive and negative deviations from its fundamental equilibrium (Baum *et al.* 2001) by its inverted bell-shaped distribution around zero. Despite the potential usefulness of the ESTAR model in modelling the exchange rate specifically and other non-linearities in general, much more empirical work has yet to be done to fill up the related literature. To this end, there are only few published articles on the STAR model. Moreover, most of them are theoretical in nature and the application is only for illustration purposes. Earlier works, for instance by Chan and Tong (1986), Luukkonen *et al.* (1988), Saikkonen and Luukkonen (1988), Luukkonen (1990), Teräsvirta (1994) and Eirtheim and Teräsvirta (1996) discuss the theoretical issues on the linearity tests and model specification of the STAR models. One notable

exception is Teräsvirta and Anderson (1993), which evaluates the forecast performance of ESTAR model, in the context of business cycles.

## PRELIMINARY DATA ANALYSIS

The data used in this paper are end-of-period nominal bilateral exchange rate for the Malaysian Ringgit vis-à-vis the Japanese Yen (RM/YEN) and relative price ( $P_t$ ), which is constructed as the ratio Consumer Price Index (CPI) of Malaysia to CPI of Japan. The data are mainly from International Monetary Fund's *International Financial Statistics* (IMF/IFS), comprising of seasonally unadjusted observations. Our sample period ranges from 1980:1 to 2000:2. The full sample period is divided into two portions. The first sub-period, which starts from 1980:1 and ends in 1997:2 is used for the model estimation purpose while the remaining observations are kept for assessing the out-sample forecast performance of the studied models<sup>2</sup>.

To test whether RM/YEN rate exhibits mean reverting behaviour to its long-run PPP equilibrium, we check for the cointegrating relationship between the two price series. However, prior to any cointegration test, the series involved should be tested for stationarity and order of integration beforehand. This is important as only variables of the same order of integration may provide a meaningful relationship. The commonly used Augmented Dickey-Fuller (ADF) and non-parametric Philips-Perron (PP) unit root tests are employed for this purpose. The results of the unit root tests as summarised in Table 1 strongly suggest that the variables are first difference stationary, which implies they are all integrated of the same order, that is,  $I(1)$ . These results hold whether trend or without trend.

Next, we proceed to investigate whether or not the long-run PPP conditions hold using the Johansen and Juselius (1990) multivariate cointegration test. Results of the trace test are depicted in Table 2. The test result provides strong evidence that RM/YEN rate and relative price are cointegrated at standard significance levels, thereby verifying that RM/YEN rate exhibits mean reverting behaviour to its long-run PPP equilibrium. The results so far obtained

<sup>2</sup> The estimation period ends in 1997: 2 and the forecasting horizon includes the 1997/98 Asian financial crises. The volatility of the exchange rates during the currency turmoil allows us to evaluate the robustness of our forecasts during the crisis and post-crisis periods.



TABLE 1  
Unit root tests results

Tests	Intercept Without Trend				Intercept with Trend			
	X	$\Delta X$	P	$\Delta P$	X	$\Delta X$	P	$\Delta P$
ADF	-0.362	-4.958*	1.989	-5.537*	-2.696	-4.953*	-0.040	-6.260*
PP	-0.528	-8.653*	3.726	-11.67*	-2.987	-8.653*	0.233	-12.86*

Notes: ADF and PP refer to Augmented Dickey-Fuller test and Phillip-Perron test respectively. X and P denote exchange rate and relative price respectively. Variable with  $\Delta$  in front means its first difference. Test-statistics with asterisk (\*) imply reject null hypothesis of unit-root at 1% significance level.

TABLE 2  
Johansen and Juselius cointegration test result

Optimal Lag	Likelihood Ratio of Eigen Value	
	$r = 0$	$r \leq 1$
10	24.369*	5.061
Critical Values		
5%	19.90	9.24
1%	24.60	12.97

Notes:  $r$  denotes the hypothesized number of cointegrating equation. Optimum lag-length is determined by the Akaike Information Criterion (AIC). The single asterisk (\*) denotes rejection of hypothesis at 5% significance level.

are consistent with those reported in Baharumshah and Ariff (1997) and M. Azali *et al.* (2001).

This finding enables us to estimate the equilibrium values of RM/YEN rate based on the PPP hypothesis. Deviations of RM/YEN rate from its equilibrium ( $y_e$ ) can then be deduced by subtracting its observed values from the estimated equilibrium values. The nature of adjustment process of these deviations towards the equilibrium position is not known yet. To determine the linearity (or non-linearity) of this adjustment process, we employ the linearity tests against the STAR models as described below.

#### LINEARITY TESTS

The minimum requirement for the estimation of STAR models is to reject the linearity of the variable under study (Tong and Lim 1980). Various linearity tests have been developed based on the idea of testing the null hypothesis that all  $\beta^*$  s in Equation 2 are simultaneously zero, against the alternative hypothesis that at least one  $\beta^*$  is not zero. Note that if the null hypothesis cannot be rejected, Equation 2 would simply be

reduced to the linear AR (p) model. By the same token, rejection of null hypothesis implies the presence of non-linearity in favour of STAR (p) model. As the properties of the transition parameter ( $\gamma^2$ ), the coefficients of non-linear terms ( $\beta^*$  s) and the mean value ( $\mu$ ) of the variable under estimation are not identified under the null hypothesis, linearity is tested in the context of auxiliary model instead of the original STAR specification as in Equation 2. Theoretical issues on linearity tests against STAR models are found in Luukkonen *et al.* (1988), Saikkonen and Luukkonen (1998), Teräsvirta and Anderson (1993), Teräsvirta (1994) and Eirtheim and Teräsvirta (1996). Interested readers may refer to these articles for more detailed discussion on the tests.

This study only highlights a specification of linearity test with alternative hypothesis in favour of the ESTAR model, a variant of STAR model relevant to this study. This specification as proposed by Teräsvirta (1994), is based on the following auxiliary regression:



$$y_t = \alpha_0 + \sum_{i=1}^p a_i y_{t-i} + \sum_{i=1}^p \sum_{j=1}^p \alpha_{ij}^* y_{t-i} y_{t-j} + \sum_{i=1}^p \sum_{j=1}^p \tau_{ij}^* y_{t-i} y_{t-j}^2 + \omega_t \quad (6)$$

The null hypothesis to be tested is that:

$$H_0 : \alpha_{ij}^* = \tau_{ij}^* = 0; i, j = 1, \dots, p \quad (7)$$

In practice, Teräsvirta's Lagrange Multiplier (LM) linearity tests can be performed by following these steps:

(1) Regress  $y_t$  on  $\{1, y_{tj}; j = 1, \dots, p\}$ . Obtain the estimated residuals  $\hat{\epsilon}_t$  and compute the residual

sum of squares,  $SSR_0 = \sum_{i=1}^T \hat{\epsilon}_i^2$ , where  $T$  is the sample size;

(2) Regress  $\hat{\epsilon}_t$  on  $\{1, y_{tj}, y_{tj}^2, y_{tj} y_{tj'}\}$ . Obtain the estimated residuals  $\hat{\omega}_t$  and compute the residual

sum of squares  $SSR = \sum_{i=1}^T \hat{\omega}_i^2$ ;

(3) Compute the test statistic:

$$LM = \frac{(SSR_0 - SSR)}{\hat{\sigma}_\epsilon^2} \quad \text{where } \hat{\sigma}_\epsilon^2 \text{ is the}$$

estimated variance of  $\hat{\epsilon}_t$  (8)

Under the null hypothesis the LM statistic is asymptotically distributed as a chi-squared ( $\chi^2$ ) with  $2p$  degrees of freedom, given that the delay parameter  $d$  is known. For unknown  $d$ , the degrees of freedom would be as large as  $0.5p(p+1)+2p$ . This LM linearity test is actually similar to that of Luukkonen, Saikkonen and Teräsvirta (LST) (1988), which is given as:

$$LST = \frac{T(SSR_0 - SSR)}{SSR_0} \quad (9)$$

LST is also asymptotically distributed as a  $\chi^2$  with  $0.5p(p+1)+p$  degrees of freedom. Luukkonen *et al.* (1988) point out that if the delay parameter  $d$  in Equation 2 is assumed known, the number of degrees of freedom for LST statistic would reduce largely to  $p+1$  only. This shows that prior knowledge about  $d$  is thus very useful in testing linearity against ESTAR model.

Briefly, the optimum lag length  $p$  in the first step of the above auxiliary regression procedure

is usually unknown even if the true model is linear, and it has to be determined from the data. Model selection criteria such as Final Prediction Error (FPE), Schwarz Information Criterion (SIC) and Akaike's Information Criterion (AIC) are normally used for this purpose. However, these criteria are of course not without any shortcomings (see for instance, Liew and Shitan (2002) for a brief review of the properties of these selection criteria). In general, these information criteria tend to penalise high-order lags.

On the other hand, if the selected  $p$  is too low, the estimated AR ( $p$ ) model may suffer from autocorrelated residuals. Teräsvirta and Anderson (1993) pointed out that neglecting the autocorrelation structure of the residuals may lead to false rejection of the linearity hypothesis in favour of the non-linearities alternative, because often the test also has low power against serially correlated errors. As such, one may think that over-parameterisation of the linear AR ( $p$ ) is preferable to under-parameterisation. However, selecting a maximum lag-length greater than the true order  $p$  may also weaken the power of the test compared to the case where the maximum lag is known (Teräsvirta and Anderson 1993). Thus, it is important to select order  $p$  sufficient enough to eliminate autocorrelation.

In this study, the optimal lag length  $p$  of linear AR ( $p$ ) model is selected based on the Akaike's biased Corrected Information Criterion (AICC). This criterion selects the minimum AICC model, among a class of models with no serial correlation. Liew and Shitan (2002) examine the behaviour of AICC through a simulation study and find that it has little tendency to underestimate the true order  $p$ . Thus the use of AICC avoids the problem of too parsimonious model being selected. Based on AICC, the optimal  $p$  is determined as 2 in the present case.

Having selected  $p$ ,  $d$  needs to be determined. In order to specify  $d$ , linearity test is carried out for the range of values considered appropriate, in this case,  $1 \leq d \leq 5$ . If the linearity is rejected for more than one value of  $d$ , then  $d$  is determined such that  $Z(\hat{d}) = \sup Z(d)$  for  $1 \leq d \leq 5$  where  $Z$  is the selected test LM or LST. The argument behind this rule of maximising the test statistic is that the test has maximum power if  $d$  is chosen correctly, whereas an incorrect choice of  $d$  weakens the power of the test. Ljung-Box



portmanteau Q test is also employed to confirm the absence of serial correlation up to 20 lags.

Results of the linearity tests are summarised in Table 3. It is clear from the table that linearity is rejected at 1% significance level for the deviations of RM/YEN rate from the equilibrium value and hence in favour of the ESTAR model. The optimal values of  $p$  and  $d$  are determined as 2 and 1 respectively. The Q statistic suggests that the combination of  $p$  and  $d$  selected for the model yields residuals that are free from autocorrelation problem up to 20 lags. This implies ESTAR (2) process with a delay parameter,  $d=1$  is the appropriate representation of the adjustment of deviations towards the long-run PPP equilibrium for the RM/YEN rate during the sample period.

**ESTIMATED MODELS**

In this study, two versions of ESTAR model, namely the unrestricted and restricted ESTAR models are estimated for  $y_t$ , the deviations of RM/YEN rate. The unrestricted ESTAR model, is actually the model as specified in Equation 2 with a transition function given in Equation 3. The word "unrestricted" is given to differentiate it from a special case whereby certain restrictions

$$\left( \sum_{i=1}^p \beta_i = 1 \beta_i^* = -\beta_i \text{ and (Taylor and Peel 2000)} \right)$$

are imposed on it, thus resulting in a so-called restricted ESTAR model. The results obtained from these models are reported in Table 4.

Several features for the estimated unrestricted model are noteworthy here: First, the non-linear parameters ( $\beta_1^*$  and  $\gamma$ ) of the unrestricted ESTAR (2) model are statistically

TABLE 3  
Linearity tests results

Test statistics	Sup Z (d)	Delay parameter, $d$	1% Critical value
LM test	19.627	1	$\chi^2(4) = 13.28$
LST test	19.920	1	$\chi^2(3) = 11.34$
Ljung-Box Q statistic	15.912	-	$\chi^2(20)=37.57$

Notes: Null hypothesis,  $H_0$ : Linear model is correct. Rejection of  $H_0$  implies nonlinearity in favour of ESTAR model.

TABLE 4  
Estimated ESTAR (2) models

Parameters	Estimated Values (Standard Errors)	
	Unrestricted model	Restricted model
$\beta_0$	- 0.031(0.19)	-0.025 (0.02)
$\beta_1$	1.799 (0.33)**	1.442 (0.19)**
$\beta_2$	-0.340 (0.23)	-0.253 (0.16)
$\beta_1^*$	-1.170 (0.40)**	-
$\beta_2^*$	0.349 (0.50)	-
$\gamma$	1.364 (0.25)**	0.357 (0.05)**
$\hat{\sigma}_y^2$	0.171	0.171
Diagnostic Tests (Marginal Significance Values)		
$\hat{\sigma}_{ESTAR}^2$	0.002	0.002
$\hat{\sigma}_{ESTAR}^2 / \hat{\sigma}_{AR}^2$	0.769	0.819
Q (20)	17.522 [0.619]	20.492 [0.428]
WHITE	5.276 [0.809]	8.270 [0.507]
ARCH (4)	1.563 [0.816]	1.595 [0.809]
GARCH (1, 1)	0.546 [0.761]	0.531 [0.767]
LR (3)	-	8.952 [0.030]
Adjusted R <sup>2</sup>	0.882	0.878



significant at 1% level. Second, the residual variance ratio of this unrestricted ESTAR (2) model to the linear AR (2) model is 0.769 indicating that the former has a much smaller variance. This implies that the non-linear model has the ability to produce smaller forecast errors than the linear model. Third, the model passed a battery of diagnostic tests at conventional significance levels. Fourth, the adjusted  $R^2$  value (0.882), the explanatory power of this non-linear model on the adjustment of deviations, is fairly high. Fifth, the sum of linear parameters  $\beta_1$  and  $\beta_2$  equals  $1.459 > 1$ , suggesting that  $y_t$  exhibits unit root behaviour and therefore linear AR (2) itself is inadequate representation of  $y_t$ . On the

other hand,  $\sum_{i=1}^2 (\beta_i + \beta_i^*) = 0.638 < 1$  implies that the requirement for global stability is met. This confirms that  $y_t$  is mean-reverting in the non-linear specification (Baum *et al.* 2001; Taylor and Peel 2000). Lastly, our results seem to be supportive of the following relationships:  $\beta_1^* = -\beta_1$  and  $\beta_2^* = -\beta_2$ .

The conclusion that can be drawn from the first five findings is that the above non-linear model is appropriate representation of  $y_t$ , the deviations of RM/YEN rate; whereas the two last results enable us to estimate the restricted ESTAR model, with the following restrictions imposed on the unrestricted model:  $\beta_1 + \beta_2 = 1$ ,  $\beta_1^* = -\beta_1$  and  $\beta_2^* = -\beta_2$  (Taylor and Peel 2000). We utilised

the likelihood ratio (LR) test to determine the validity of these restrictions. The computed LR statistic is compared with chi-squared critical value with 3 degrees of freedom. The LR statistic of 8.952 suggests that the above restrictions cannot be rejected at the 5% significance level. The adequacy of the model is verified by the absence of serial correlation (Ljung-Box Q statistic) and heteroscedasticity (WHITE statistic). Moreover, neither ARCH (4) nor GARCH (1, 1) suggests the presence of ARCH effect. Thus, the restricted ESTAR (2) model passes a battery of diagnostic tests and thereby can be used as a forecasting model, as its unrestricted version.

The linear AR model is also estimated for the purpose of forecast accuracy comparison. The estimation of AR model requires that the variables must be stationary; otherwise interpretation from the outcome would be spurious. We employed classical ADF and PP stationarity tests to check whether the stationary requirement is met. The stationarity tests results are summarized in Table 5. The results in Table 5 postulate that instead of  $y_t$ , which is not stationary, we should estimate linear AR model for stationary series,  $\Delta y_t$ , the first difference of  $y_t$ . Strategically, we need to estimate AR (1) model for  $\Delta y_t$  in order to obtain the required benchmark model, namely AR (2) model for  $y_t$ . The estimated AR (1) model is reported in Table 6.

TABLE 5  
Stationarity tests results

Tests	Intercept Without Trend		Intercept with Trend	
	$y_t$	$\Delta y_t$	$y_t$	$\Delta y_t$
ADF	-0.858	-4.081*	-0.034	4.081*
PP	-1.244	-8.657*	-0.493	-8.627*

Note: \* denotes the variable is stationary at 1% significance level.

TABLE 6  
Estimated AR (1) model

Series	Coefficient of		Diagnostic Tests (Marginal Significance Values)			
	$\Delta y_{t-1}$ (t statistic)	$R^2$	Q (20)	WHITE	ARCH (4)	GARCH
$\Delta y_t$	0.024 (-0.96)	0.857	16.919 (0.657)	2.267 (0.757)	1.149 (0.362)	0.037 (0.982)



From Table 6, our AR (1) model for  $\Delta y_t$  is given by:

$$\Delta y_t = - 0.024 \Delta y_{t-1} \quad (10)$$

Diagnostic tests results show that Equation 10 is free from all the autocorrelation, heteroscedasticity and ARCH effects. The adjusted  $R^2$  values also suggest that the AR (1) model fits fairly well for the  $\Delta y_t$  series. To sum up, the estimated AR (1) model is an appropriate representation of first difference series of  $y_t$ . The only shortcoming of this model is that the linear autoregressive parameter is not significant based on the standard t-test. Equation 10 can be rewritten as:

$$y_t = 0.976 y_{t-1} + 0.024 y_{t-2} \quad (11)$$

which is exactly the AR (2) specification of  $y_t$ , the deviation of MYR/JPY rate from its PPP equilibrium position. If we take into account the insignificance of the autoregressive parameter, Equation 10 would effectively be reduced to:

$$\Delta y_t = 0 \quad (12)$$

or its equivalent:

$$y_t = y_{t-1} \quad (13)$$

Equation 13 is simply the commonly used benchmark for the evaluation of exchange rate forecasting models, namely the simple random walk (SRW) models. With the availability of two benchmarks model, in particular the linear AR (2) model (Equation 11) and SRW model (Equation 13), this study proceeds to compare the forecast accuracy of the unrestricted and restricted ESTAR (2) models with the benchmark (AR(2) and SRW) models.

### FORECAST PERFORMANCE

The models are used to generate out-sample forecasts and the forecasting performances of these models are evaluated. The out-sample performance of the estimated forecasting models over the forecast horizon of  $n = 4, 8$  and  $12$  quarters during the period 1997:3 to 2000:2 are evaluated based on mean absolute forecast error (MAFE), mean absolute percentage forecast error (MAPFE) and root mean square forecast error (RMSFE). The overall forecasting performances are reported in Table 7. Generally, all accuracy criteria consistently suggest that all three forecasting models, namely the unrestricted ESTAR (2), restricted ESTAR (2) and AR (2) models outperformed the SRW model at all horizons. This implies that both the linear and non-linear time series models under this study improve over the SRW model in the short- and

TABLE 7  
Comparison of out-sample forecast accuracy

Models	Unrestricted ESTAR (2)			Restricted ESTAR (2)			AR (2)		
	4	8	12	4	8	12	4	8	12
Simple Random Walk model as benchmark									
MAFE	0.721	0.919	0.954	0.671	0.880	0.934	0.721	0.919	0.954
Ratio									
MAFPE	0.587	0.838	0.890	0.668	0.876	0.923	0.712	0.901	0.935
Ratio									
RMSPE	0.731	0.846	0.851	0.804	0.903	0.908	0.825	0.950	0.962
Ratio									
Linear AR (2) model as benchmark									
MAFE	0.590	0.844	0.903	0.931	0.958	0.979			
Ratio									
MAFPE	0.824	0.930	0.952	0.938	0.972	0.988			
Ratio									
RMSPE	0.886	0.891	0.885	0.975	0.950	0.944			
Ratio									



medium-run.

It is interesting to know whether the non-linear models yield more accurate forecasts than the linear model. As a matter of fact, it is rational for one to expect the former to improve upon the latter, since the former is more involved. In view of the fact that the actual answer to this issue is important to the application of time series analysis, this empirical study proceeds further to contrast the forecast performance between these two competing models directly. The conclusion from this exercise is that both the unrestricted and restricted ESTAR (2) models predict better than linear AR (2) model on the basis of all accuracy criteria, for forecast horizon equals 4, 8 and 12 quarters. For instance, MAFE of unrestricted (restricted) ESTAR (2) model are 0.590 (0.931), 0.844 (0.958) and 0.903 (0.979) times smaller than the MAFE of AR (2) model, for  $n = 4, 8$  and 12, in that order. These results are overwhelmingly supported by the MAFPE and RMSPE criteria. As such, our extra resource spent on the modelling of ESTAR model is at least paid off. Perhaps, more importantly, this study has provided evidence that the performance of ESTAR is superior to its linear competitor, the AR model.

### CONCLUSION

The empirical performance of foreign exchange rate models has been frequently criticized in recent years. These critiques come from studies that have found exchange rate models predict poorly out of sample periods. In the foreign exchange market, central banks often intervene, in an effort either to attenuate or to amplify variations in the exchange rate. The Ringgit is no exception and this explained partly the poor-out-of sample prediction of the exchange rate models in previous studies.

In this study, we demonstrate that the adjustment of the RM/YEN rate is in fact predictable based on time series model. The STAR model and AR model both outperform the random walk in the out sample forecasting at all horizons. The results also show that the STAR model outperforms the AR model, its linear competitor. Importantly, we demonstrate formally that the adjustment of the Ringgit to its long run equilibrium follows a non-linearity path. This suggests that there is a systematic predictable component in the movement of nominal RM/

YEN exchange rate.

Two major implications of this finding are: (1) non-linear ESTAR model should be given priority in modelling exchange rate time series; and (2) exchange rate forecasters could rely on the non-linear model as a more reliable forecasting tool.

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